# PLANNED INSTRUCTION

## A PLANNED COURSE FOR:

## Mathematical Modeling

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**Grade Level: 11, 12**

**Date of Board Approval: 2024**

## Course Weighting: Mathematical Modeling

| | |
|---|---|
| **Weekly Assessments** | **50%** |
| **Projects** | **40%** |
| **Homework/Classwork** | **10%** |
| **Total** | **100%** |

# Curriculum Map

**Overview:**

This class is a collection of mathematical situations drawn from real life. It is a class that will foster collaborating with classmates, working as a team member and drawing conclusions from data. Appropriate technology through the use of graphing calculators and other technology based graphing utilities will be accessed to model and analyze data. Students will explore problem solving skills, linear function models, quadratic function models, exponential function models, geometric models, statistical and graphical models, financial models, and probability models.

**Time/Credit for the Course:** One full year, meeting daily for ~46 minutes, 1 credit

**Goals:**

**Understanding of:**
**Marking Period One: Unit 1 Linear Functions, Systems of Equations & Inequalities**
**Overview based on 45 days**

- Applications of linear models
- Constructing, analyzing, comparing, and explaining linear relationships in real life scenarios
- Determining validity and precision of mathematical models utilizing technology
- Solving systems of linear equations and inequalities as a mathematical model
- Using augmented matrices to solve systems of linear equations
- Solving a real-life situation using linear programming

**Marking Period Two: Unit 2 Quadratics & Family of Functions**
**Overview based on 45 days**

- Graphs of quadratic functions
- Methods to solve quadratic functions
- Writing quadratic functions
- Systems of linear and quadratic functions
- Applications of quadratic models
- Constructing, analyzing, comparing, and explaining quadratic relationships in real life scenarios
- Determining validity and precision of mathematical models utilizing technology
- Identifying Functions: linear, quadratic, absolute value, square root, constant, cubic, cube root, and reciprocal function
- Transformations of functions by hand and with a graphing utility
- Mathematical models and applications of functions

**Marking Period Three: Unit 3 Exponential, Logarithmic, & Piecewise Functions**
**Overview based on 45 days**

- Graphs of exponential functions
- Writing exponential functions
- Evaluating exponential functions
- Using inverse operations to solve exponential equations for variables
- Exponential growth and decay
- Applications of exponential models
- Constructing, analyzing, comparing and explaining exponential relationships in real life scenarios
- Determining validity and precision of mathematical models utilizing technology
- Graphing, writing, and evaluating values of piecewise functions
- Mathematical models and applications of piecewise functions

**Marking Period Four: Unit 4: Modeling: Financial, Statistical, & Geometrical**
**Overview based on 45 days**

- Measures of central tendencies and variability
- Personal finances

- Calculating and comparing loan options
- Analysis of data displays (bar graphs, stem and leaf plots, line plots)
- Organizing data with tables and graphs
- Applications of Pythagorean Theorem
- Trigonometric ratios (sine, cosine and tangent ratios)
- Applications involving trigonometry
- Area versus perimeter of polygons
- Volume and surface area of 3-dimensional figures
- Applications of geometric formulas to solve problems
- Scale factors and proportional relationships

**Big Ideas:**

**Big Idea # 1:** Mathematical relationships can be represented as expressions, equations and inequalities in mathematical situations.

**Big Idea # 2:** Mathematical relations and functions can be modeled through multiple representations and analyzed to raise and answer questions.

**Big Idea # 3:** Data can be modeled and used to make inferences.

**Big Idea # 4:** Patterns exhibit relationships that can be extended, described, and generalized.

**Big Idea # 5:** Mathematical relationships among numbers can be represented, compared, and communicated.

**Textbook and Supplemental Materials and Resources:**
**Primary Textbook(s) Used for this Course of Instruction**
Name of Textbook: *Applied Mathematics for College and Career*
Textbook ISBN #: 978-0-13-405940-2
Textbook Publisher &Year of Publication: Pearson Inc. 2016

**Supplemental Resources:**
- Mathmedic worksheets
- Graphing Calculator: TI-84 and Desmos
- TI Smart View Software
- Teacher developed SAT and Keystone question bank

# Curriculum Plan

**Unit 1:** Linear Functions, Systems of Equations & Inequalities **Time/Days:** 45 days

**Standards (by number):**
CC.2.1.HS.F.3, CC.2.1.HS.F.4, CC.2.1.HS.F.5, CC.2.2.HS.C.1, CC.2.2.HS.C.2, CC.2.2.HS.C.3, CC.2.2.HS.C.5, CC.2.2.HS.C.6, CC.2.2.HS.D.7, CC.2.2.HS.D.8, CC.2.2.HS.D.9, CC.2.2.HS.D.10

**Anchors:**
A1.1.2.1, A1.1.2.2, A1.1.3.1, A1.1.3.2, A1.2.1.1, A1.2.1.2, A1.2.2.1, A2.2.3.1

**Eligible Content:**
A1.1.2.1.1 Write, solve and/or apply a linear equation (including problem situations).
A1.1.2.1.2 Use and/or identify an algebraic property to justify any step in an equation solving process (linear equations only).
A1.1.2.1.3 Interpret solutions to problems in the context of the problem situation (linear equations only).
A1.1.2.2.1 Write and/or solve a system of linear equations (including problem situations) using graphing, substitution and/or elimination (limit systems to 2 linear equations).
A1.1.2.2.2 Interpret solutions to problems in the context of the problem situation (systems of 2 linear equations only)
A1.1.3.1.1 Write or solve compound inequalities and/or graph their solution sets on a number line (may include absolute value inequalities).
A1.1.3.1.2 Identify or graph the solution set to a linear inequality on a number line.
A1.1.3.1.3 Interpret solutions to problems in the context of the problem situation (limit to linear inequalities).
A1.1.3.2.1 Write and/or solve a system of linear inequalities using graphing (limit systems to 2 linear inequalities).
A1.1.3.2.2 Interpret solutions to problems in the context of the problem situation (systems of 2 linear inequalities only).
A1.2.1.1.1 Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
A1.2.1.1.3 Identify the domain or range of a relation (may be presented as ordered pairs, a graph, or a table).
A1.2.1.2.1 Create, interpret and/or use the equation, graph or table of a linear function.
A1.2.1.2.2 Translate from one representation of a linear function to another (graph, table and equation).
A1.2.2.1.1 Identify, describe and/or use constant rates of change.
A1.2.2.1.2 Apply the concept of linear rate of change (slope) to solve problems.
A1.2.2.1.3 Write or identify a linear equation when given the graph of the line, 2 points on the line, or the slope and a point on a line. (Linear equations may be in point-slope, standard and/or slope-intercept form).
A1.2.2.1.4 Determine the slope and/or y-intercept represented by a linear equation or graph.
A2.2.3.1.1 Draw, identify, find and/or write an equation for a regression model (lines and curves of best fit) for a scatter plot.

A2.2.3.1.2 Make predictions using the equations or graphs of regression models (lines and curves of best fit) of scatter plots.

**Objectives:**
Students will be able to

1. Practice communication skills and write a solution in sentences. (DOK 1)
2. Organize information and develop problem-solving skills (draw a picture, recognize a pattern, do a simpler problem). (DOK 2)
3. Identify input and output in situations involving two variable quantities and identify a functional relationship between two variables. (DOK 1)
4. Identify independent and dependent variables and use a table to numerically represent a functional relationship between two variables. (DOK 1)
5. Represent a functional relationship graphically and identify trends in data pairs. (DOK 1)
6. Identify and interpret reasonable domain and range for linear functions to model real life situations. (DOK 2)
7. Represent and solve linear functions numerically and graphically using technology. (DOK 3)
8. Identify and develop a mathematical model in the form of an equation and use the model to solve problems. (DOK 4)
9. Recognize patterns between two variables and develop a mathematical model to solve a problem. (DOK 2)
10. Use problem solving skills to make decisions based on solutions of mathematical models. (DOK 4)
11. Determine the average rate of change and interpret slope, including zero and undefined slopes. (DOK 2)
12. Identify whether a situation can be modeled by a linear function and determine the practical meaning of the x and y intercepts. (DOK 3)
13. Write the equation of a line from different sets of given information in slope intercept form, in point slope form and in standard form. (DOK 2)
14. Determine the equation of a horizontal or vertical line. (DOK 2)
15. Create equations, graphs, and tables of linear functions. (DOK 1 and DOK 2)
16. Construct scatterplots from sets of data pairs and identify whether a positive, negative or no correlation exists. (DOK 3)
17. Estimate and draw a line of best fit through a set of points in a scatterplot. (DOK 2)
18. Use graphing calculators and other online resources to graph data and produce a line of best fit. (DOK 2)
19. Collect and organize data in a table, plot data in a scatterplot, recognize linear patterns, and determine a linear regression equation. (DOK 4)
20. Explain interpolations and extrapolations with a linear function and the meaning in real world scenarios. (DOK 3)
21. Solve a system of two linear equations numerically, graphically, using the substitution method, or the elimination method. (DOK 2)
22. Interpret the solution to a system of two linear equations in terms of the problem's content. (DOK 3)
23. Determine the break-even point of a linear system algebraically and graphically and interpret break-even points in contextual situations. (DOK 2)

24. Use an augmented matrix to solve a system of equations with two or more variables, including systems with no solution or systems with infinite solutions. (DOK 3)
25. Mathematically model a real-world situation using a system of linear equations and to solve that problem using any of the following methods: Graphing, substitution, elimination, or augmented matrices. (DOK 4)
26. Solve linear inequalities including compound inequalities algebraically and graphically. (DOK 2)
27. Solve a system of linear inequalities in two variables graphically. (DOK 2)
28. Model a real-world situation using linear programming by developing an objective function and its constraints, graphing the feasible region, and finding the points of intersection of these constraints which will be the maximum and minimum values of the objective function. (DOK 4)

**Core Activities and Corresponding Instructional Methods:**

1. Develop students' collaboration skills and expose their prior knowledge and current mathematical abilities.
    a. Instructional Methods:
        i. Diagnostic questioning
        ii. Cooperative learning groups
    b. Core Activities:
        i. Numeracy tasks
2. Develop students' skills in writing, graphing, analyzing, and interpreting linear functions and inequalities in real world scenarios.
    a. Instructional Methods:
        i. Diagnostic questioning and exploration of linear functions and inequalities
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
    b. Core Activities:
        i. Arm Length vs. Height Project- The height and wingspan of students will be measured. By hand, students will graph the data by labeling and determining the scale for the x and y axis. Students will draw a line of best fit and create an equation for the line. The data will be inputted into excel where students will graph the data, find the r-squared value, and compare their line of best fit to excel line of best fit.
        ii. Pasta Bridge Project- Given pennies, a cup, and pasta, students will create pasta bridges to collect and record the relationship between the number of strands of pasta versus the number of pennies the cup can hold. By hand, students will graph the data by labeling and determining the scale for the x and y axis. Students will draw a line of best fit and create an equation for the line. The data will be inputted into excel where students will graph the data, find the r-squared value, and compare their line of best fit to excel line of best fit.
        iii. Egg Drop Project - Students will produce a simulation of bungee jumping given rubber bands, an egg, and meter sticks. Students will collect and record the relationship between the number of rubber bands versus the distance the egg drops. By hand, students will graph the data by labeling

and determining the scale for the x and y axis. Students will draw a line of best fit and create an equation for the line. The data will be inputted into excel where students will graph the data, find the r-squared value, and compare their line of best fit to excel line of best fit. Given their model or the model produced by excel, students will then predict how many rubber bands will be needed to drop the egg off the stadium of the school to get closest to the ground without breaking the egg.

3. Develop students' skills in solving real world systems of linear equations by graphing, using the substitution method, using the elimination method, or using an augmented matrix.
    a. Instructional Methods:
        i. Diagnostic questioning and exploration of systems of linear equations
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
    b. Core Activities:
        i. Applications of Systems of Equations- Given the real world scenarios, students will define variables, write equations, and choose a method to solve the system of equations. Students will interpret their solutions.
        ii. Business Math- Define revenue function, cost function, profit function, and the break-even point. Practice writing the linear functions, using the function to answer questions, finding the break-even point using substitution, graphing the functions, and interpreting the solutions.
4. Develop students' skills in graphing systems of linear inequalities and using linear programming to find the maximum and minimum of an objective function given linear constraints given a real world situation.
    a. Instructional Methods:
        i. Diagnostic questioning and exploration of systems of linear inequalities
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
    b. Core Activities:
        i. Linear Programming - Given the real world scenarios, students will practice writing objective functions and inequalities for the constraints. By graphing the constraints by hand or using a Desmos calculator students will find the feasible region and the points of intersection of the constraints. Students will use these points and the objective function to find maximum and minimum values. Students will interpret these solutions.
5. Weekly Cumulative Assessments

**Assessments:**

- **Diagnostic:**
    - Numeracy tasks on vertical whiteboards and in small groups

    - Teacher questioning and observation
    - Algebra 1 Keystone Exam
- **Formative:**
    - Teacher observations, questions, discussions
    - Individual and group practice
    - Teacher prepared projects
    - Teacher prepared weekly cumulative assessments
- **Summative:**
    - Teacher Prepared Weekly Cumulative Assessments

**Extensions:**

- Desmos Linear Functions Practice
- Pasta Bridge Project: Compare and analyze the effect of using different pastas. Examples: angel hair, spaghetti, fettuccine, linguine
- Egg Drop Project: Measuring elasticity of rubber bands before and after the drop
- Wonderlic Practice Test - Wonderlic Contemporary Cognitive Ability Test is an assessment used to measure the cognitive ability and problem-solving aptitude of prospective employees for a range of occupations. The test consists of 50 multiple choice questions to be answered in 12 minutes. Math, vocabulary and reasoning are skills that are tested.

**Correctives:**

- Remediation: IXL and/or Delta math
- Desmos Activities
- Practice worksheets generated through Kuta Software

**Unit 2:** Quadratics & Family of Functions **Time/Days:** 45 days

**Standards (by number):**
CC.2.1.HS.F.3, CC.2.1.HS.F.4, CC.2.2.HS.C.1, CC.2.2.HS.C.2, CC.2.2.HS.C.3, CC.2.2.HS.C.4, CC.2.2.HS.C.5, CC.2.2.HS.C.6, CC.2.2.HS.D.2**,** CC.2.2.HS.D.7, CC.2.2.HS.D.8, CC.2.2.HS.D.9

**Anchors:**
A2.1.3.1, A2.1.3.2, A2.2.1.1, A2.2.2.1, A2.2.2.2, A2.2.3.1

**Eligible Content:**
A2.1.2.2.1 Factor algebraic expressions, including difference of squares and trinomials (trinomials limited to the form ax2+bx+c where a is not equal to 0).
A2.1.3.1.1 Write and/or solve quadratic equations (including factoring and using the Quadratic Formula).
A2.1.3.2.1 Determine how a change in one variable relates to a change in a second variable (e.g., y=4/x, if x doubles, what happens to y?).
A2.2.1.1.4 Identify and/or determine the characteristics of an exponential, quadratic, or polynomial function (e.g., intervals of increasing/decreasing, intercepts, zeros, and asymptotes).
A2.2.2.1.1 Create, interpret and/or use the equation, graph or table of a polynomial function (including quadratics).
A2.2.2.1.3 Determine, use and/or interpret minimum and maximum values over a specified interval of a graph of a polynomial, exponential or logarithmic function.
A2.2.2.1.4 Translate a polynomial, exponential or logarithmic function from one representation to another (graph, table and equation).
A2.2.2.2.1 Identify or describe the effect of changing parameters within a family of functions.
A2.2.3.1.1 Draw, identify, find and/or write an equation for a regression model (lines and curves of best fit) for a scatter plot.
A2.2.3.1.2 Make predictions using the equations or graphs of regression models (lines and curves of best fit) of scatter plots.

**Objectives:**
Students will be able to

1. Evaluate and graph quadratic functions. (DOK 1)
2. Interpret the coordinates of points on the graph of a quadratic in context. (DOK 2)
3. Identify and interpret reasonable domain and range for quadratic functions to model real life situations. (DOK 2)
4. Solve a quadratic equation numerically, graphically and algebraically by taking square roots and using the quadratic formula. (DOK 2)
5. Write the quadratic equation in vertex form given points on the graph, the vertex, or the x-intercepts. (DOK 4)
6. Write the quadratic equation given three points on the function using a system of equations and matrices. (DOK 4)
7. Apply their understanding of quadratic functions to model real-world situations and solve word problems. (DOK 4)
8. Collect, graph, and analyze data from real life experiences for quadratic and linear models. (DOK 3 and DOK 4)

9. Produce and use the line of best fit given a scatter plot using graphing calculators, Desmos, or excel. (DOK 2)
10. Identify the graph of linear, quadratic, absolute value, square root, constant, cubic, cube root, and reciprocal functions. (DOK 1)
11. Create equations, graphs, and tables for the family of functions. (DOK 1 and DOK 2)
12. Interpret the coordinates of points on the graph of a function in context. (DOK 2)
13. Identify specific transformations (horizontal shift, vertical shift, stretch or compression) of a linear, quadratic, absolute value, square root, constant, cubic, cube root, and reciprocal function with and without a graphing calculator. (DOK 2)
14. Identify and interpret reasonable domain and range for the family of functions to model real life situations. (DOK 2)

**Core Activities and Corresponding Instructional Methods:**

1. Expose students' prior knowledge of functions and graphing in the coordinate plane, guiding students to graph quadratic functions. Investigate the properties of quadratic functions.
    a. Instructional Methods:
        i. Diagnostic questioning and exploration of quadratic functions.
        ii. Cooperative learning groups
    b. Core Activities:
        i. Ball Toss Project - Students will use a ball to collect data between the number of passes of the ball versus the time in seconds it took for the students to complete those passes. Students will use Desmos, excel or a graphing calculator to graph the data and find the line of best fit. Students will record the r squared value, interpret their line of best fit, and use the line of best fit to make predictions. Students will then repeat the processes, but this time they will begin saying the alphabet in between passes. Students will use Desmos, excel or a graphing calculator to graph the data and find the line of best fit. Students will compare and contrast the difference between linear and quadratic functions.
2. Develop students' ability to interpret quadratic functions using mathematical language, graph quadratics functions, and solve quadratic functions.
    a. Instructional Methods:
        i. Encouraging and guiding students to use appropriate math terminology
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
3. Develop students' ability to solve real world problems involving quadratic functions.
    a. Instructional Methods:
        i. Direct instruction using Smart Technology
        ii. Guided practice
        iii. Cooperative learning groups
    b. Core Activities:

i. Writing Quadratic Equations - Students will write quadratic equations given three points and using an augmented matrix. After writing the quadratic equations, students will be able to use the equation to find x-intercepts, y-intercept, vertex, domain, range, and evaluate given an x-coordinate. Students will interpret their solutions in context to the problem.
ii. Projectile Motion - Students will write, graph and analyze quadratic functions involving projectile motion. Students will be able to use the equation to find x-intercepts, y-intercept, vertex, domain, range, and evaluate given an x- coordinate. Students will interpret their solutions in context to the problem.
iii. How High Can You Throw? - Given meter sticks, a stopwatch, and a tennis ball, students will throw the tennis ball straight up into the air. Students will run trials and collect data on the height of the tennis ball when it left the thrower's hand and how long it took the tennis ball to reach the ground. Given this data, students will write an equation, find the maximum height of their ball, and graph the data.

4. Develop students' ability to identify and perform transformations on linear, quadratic, absolute value, square root, constant, cubic, cube root, and reciprocal function.
    a. Instructional Methods:
        i. Diagnostic questioning and exploration of transformations
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
    b. Core Activities:
        i. Transformation Exploration- Students will explore transformations of a general function using guided notes and Desmos activity builder.
        ii. Desmos Graphing Calculator- Use Desmos graphing calculator to introduce parent functions. Students will practice performing transformations.
5. Weekly Cumulative Assessments

**Assessments:**

- **Diagnostic:**
    - Vertical whiteboards and small group collaboration
    - Teacher questioning and observation
    - Algebra 1 Keystone Exam
- **Formative:**
    - Teacher observations, questions, discussions
    - Individual and group practice
    - Teacher prepared projects
    - Teacher prepared weekly cumulative assessments

- **Summative:**
    - Teacher Prepared Weekly Cumulative Assessments

**Extensions:**

- How high can you throw? Project - Comparing the heights of different objects and different throwers
- Modeling Quadratics Geometric Problems - Students will use their knowledge about quadratics, area, and volume to solve real world problems.
- Desmos Transformations Practice

**Correctives:**

- Remediation: IXL and/or Delta math
- Desmos Activities
- Practice worksheets generated through Kuta Software

**Unit 3:** Exponential, Logarithmic, & Piecewise Functions **Time/Days:** 45 days

**Standards (by number):**
CC.2.1.HS.F.3, CC.2.2.HS.C.1, CC.2.2.HS.C.2, CC.2.2.HS.C.3, CC.2.2.HS.C.4, CC.2.2.HS.C.5, CC.2.2.HS.C.6, CC.2.2.HS.D.2, CC.2.2.HS.D.7, CC.2.2.HS.D.8, CC.2.2.HS.D.10

**Anchors:**
A1.1.1.5 , A2.1.2.1, A2.1.2.2, A2.2.1.1, A2.1.3.1, A2.2.2.1, A2.2.3.1

**Eligible Content:**
A1.1.1.5.2 Factor algebraic expressions, including difference of squares and trinomials (trinomials limited to the form ax2+bx+c where a is equal to 1 after factoring out all monomial factors).
A2.1.2.1.2 Simplify/evaluate expressions involving positive and negative exponents and/or roots (may contain all types of real numbers - exponents should not exceed power of 10).
A2.1.2.1.3 Simplify/evaluate expressions involving multiplying with exponents, powers of powers and powers of products (limit to rational exponents).
A2.1.2.1.4 Simplify or evaluate expressions involving logarithms and exponents (e.g. log28 = 3 or log42 = ½).
A2.2.1.1.1 Analyze a set of data for the existence of a pattern and represent the pattern with a rule algebraically and/or graphically.
A2.2.1.1.3 Determine the domain, range or inverse of a relation.
A2.2.1.1.4 Identify and/or determine the characteristics of an exponential, quadratic, or polynomial function (e.g., intervals of increasing/decreasing, intercepts, zeros, and asymptotes).
A2.1.3.1.3 Write and/or solve a simple exponential or logarithmic equation (including common and natural logarithms).
A2.1.3.1.4 Write, solve and/or apply linear or exponential growth or decay (including problem situations).
A2.2.2.1.2 Create, interpret and/or use the equation, graph or table of an exponential or logarithmic function (including common and natural logarithms).
A2.2.2.1.3 Determine, use and/or interpret minimum and maximum values over a specified interval of a graph of a polynomial, exponential or logarithmic function.
A2.2.2.1.4 Translate a polynomial, exponential or logarithmic function from one representation to another (graph, table and equation).
A2.2.3.1.1 Draw, identify, find and/or write an equation for a regression model (lines and curves of best fit) for a scatter plot.
A2.2.3.1.2 Make predictions using the equations or graphs of regression models (lines and curves of best fit) of scatter plots.

**Objectives:**
Students will be able to

1. Define growth factor and determine the growth factor from percent increases as well as apply growth factors to problems involving percent increases. (DOK 2)
2. Define decay factor and determine the decay factor from percent decreases as well as apply decay factors to problems involving percent decreases. (DOK 2)

3. Recognize and graph an exponential function (growth or decay) from numerical data or equations. (DOK 2)
4. Determine the growth or decay factor of an exponential function represented by a table of values or an equation. (DOK 1-3)
5. Analyze the exponential function representing the given data set to determine the amount of time required to double or half the dependent variable (national debt/population). (DOK 4)
6. Solve exponential equations using common logarithms or natural logarithms. (DOK 3)
7. Compare and contrast the formulas for simple and compound interest and apply to real word applications. (DOK 3)
8. Apply the continuous compounding formula $A = Pe^{rt}$. (DOK 2)
9. Collect, graph, and analyze data from real life experiences. (DOK 1, DOK 2, and DOK 3)
10. Produce and use the line of best fit given a scatter plot using graphing calculators, Desmos, or excel. (DOK 2)
11. Use exponential or logarithmic models to solve real-world problems such as compound interest, exponential growth, and radioactive decay. (DOK 4)
12. Identify the independent and dependent variables of a model. (DOK 1)
13. Interpret story graphs and sketch the graph given a real-world situation with two related variables. (DOK 2)
14. Identify the domain and/or range of a piecewise function and to determine whether a given relation is a function. (DOK 2)
15. Find values for a piecewise function. (DOK 2)
16. Analyze the graph of a piecewise function by identifying the intercepts, zeros and extrema. (DOK 4)
17. Determine where the piecewise function is increasing, decreasing, or constant, and whether the function is even or odd (DOK 2)
18. Graph piecewise functions and write the equation of a piecewise function given the graph (DOK 2)
19. Apply their understanding of piecewise functions to model real-world situations and solve word problems. (DOK 4)

**Core Activities and Corresponding Instructional Methods:**

1. Expose students' prior knowledge of exponential functions and investigate the properties of exponential functions.
    c. Instructional Methods:
        i. Diagnostic questioning and exploration of exponential functions.
        ii. Cooperative learning groups
    d. Core Activities:
        i. M&M Project- Students will shake and pour a cup of M&Ms and remove each M&M that has the symbol showing. Students will count the leftover M&Ms and repeat the process recording their data after each shake. Students will graph their data using desmos graphing calculator and write their own equations to represent the pattern shown in their data and

discover exponential decay with a decay rate of about 50%. Students will compare and interpret their y-intercept, growth factor, and asymptote found during their simulation.

ii. Zombie Problem- Students will investigate exponential growth by investigating a town that gets taken over by zombies. Students will find the pattern, graph the data, and write equations. Students will compare and contrast exponential versus linear functions.

2. Develop students' ability to solve real world exponential problems, specifically problems involving exponential growth or decay.
    a. Instructional Methods:
        i. Direct instruction using Smart Technology
        ii. Guided practice
        iii. Cooperative learning groups
    b. Core Activities:
        i. Compound Interest- Students will use compound interest and compounded continuously interest formulas to find the amount of interest, the interest rate, or the initial amount invested. Students will use common logarithms and natural logarithms to solve exponential problems for time.
        ii. Radioactive Decay- Students will analyze and write exponential functions to solve real-world problems including half-life problems.
3. Develop students' ability to evaluate, write, and graph piecewise functions.
    a. Instructional Methods:
        i. Diagnostic questioning and exploration of piecewise functions
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
4. Develop students' ability to solve real world problems involving piecewise functions.
    a. Instructional Methods:
        i. Direct instruction using Smart Technology
        ii. Guided practice
        iii. Cooperative learning groups
    b. Core Activities:
        i. Change of Phase Exploration- Given a hot plate, ice, a beaker, and a temperature probe, students will collect data on the water's temperature every twenty seconds until the water comes to a boil and boils for at least two minutes. Students will graph this data by hand and then use desmos graphing calculator to graph the data. Using what they know about transformations of functions and piecewise functions, students will find three equations to represent the phase changes that happen when cold water is brought to a boil.
        ii. Piecewise Function Art -Students will find a unique, school appropriate picture that they will recreate using a piecewise function. Students will practice transforming functions to match their image and restricting the sub functions domain and range.
        iii. US Federal Income Tax- Students will write and analyze a piecewise function to represent the federal income tax brackets in the United States.

5. Weekly Cumulative Assessments

**Assessments:**

- **Diagnostic:**
  - Vertical whiteboards and small group collaboration
  - Teacher questioning and observation
- **Formative:**
  - Teacher observations, questions, discussions
  - Individual and group practice
  - Teacher prepared projects
  - Teacher prepared weekly cumulative assessments
- **Summative:**
  - Teacher Prepared Weekly Cumulative Assessments

**Extensions:**

- Story Graphs on Desmos- Students can practice sketching graphs based on a scenario or video
- Piecewise Function Story Graphs- Students will create a unique story and a piecewise function to match their story. Students will consider the x and y axis labels and scales. Students will have to answer questions about when they function. For example, students will find the domain, range, and when the function is increasing or decreasing.

**Correctives:**

- Remediation: IXL and/or Delta math
- Desmos Activities
- Practice worksheets generated through Kuta Software

**Unit 4:** Modeling: Financial, Statistical, & Geometrical **Time/Days:** 45 days

**Standards (by number):**
FCS Standards: 11.1.12.B, 15.1.12.H, 15.6.12.B, 15.6.12.G, 15.6.12.I, 15.6.12.P

CC.2.3.HS.A.3, CC.2.3.HS.A.5, CC.2.3.HS.A.6, CC.2.3.HS.A.7, CC.2.4.HS.B.5, CC.2.2.HS.C.9, CC.2.3.HS.A.14

**Anchors:**
A1.2.3.2, G.1.3.1, G.2.1.1, G.2.2.2, G.2.3.1

**Eligible Content:**
11.1.12.B Analyze the management of financial resources across the lifespan.
15.1.12.H Identify, calculate, and record depreciation, depletion and amortization; explain their effect on the financial statements.
15.6.12.B Analyze financial decisions for major purchasing events occurring at different stages in life, systematically considering alternatives and consequences.
15.6.12.G Identify strategies for personal financial management.
15.6.12.I Analyze the functions of the Federal Reserve and other financial institutions.
15.6.12.P Develop financial investment plans to accommodate various economic and personal scenarios.
A1.2.3.2.1 Estimate or calculate to make predictions based on a circle, line, bar graph, measures of central tendency, or other representations.
A1.2.3.2.2 Analyze data, make predictions, and/or answer questions based on displayed data (box-and-whisker plots, stem-and-leaf plots, scatter plots, measures of central tendency, or other representations).
A1.2.3.2.3 Make predictions using the equations or graphs of best-fit lines of scatter plots.
G.1.3.1.1 Identify and/or use properties of congruent and similar polygons or solids.
G.1.3.1.2 Identify and/or use proportional relationships in similar figures.
G.2.1.1.1 Use the Pythagorean Theorem or trigonometric ratios to write and/or solve problems involving right triangles.
G.2.1.1.2 Use trigonometric ratios to write and/or solve problems involving right triangles.
G.2.2.2.1 Estimate area, perimeter, or circumference of an irregular figure.
G.2.2.2.2 Find the measurement of a missing length given the perimeter, circumference, or area.
G.2.2.2.3 Find the side lengths of a polygon with a given perimeter to maximize the area of the polygon.
G.2.2.2.4 Develop and/or use strategies to estimate the area of a compound/composite figure.
G.2.2.2.5 Find the area of a sector of a circle.
G.2.3.1.1 Calculate the surface area of prisms, cylinders, cones, pyramids and/or spheres. Formulas are provided on the reference sheet.
G.2.3.1.2 Calculate the volume of prisms, cylinders, cones, pyramids and/or spheres. Formulas are provided on the reference sheet.
G.2.3.1.3 Find the measurement of a missing length given the surface area or volume.

**Objectives:**

Students will be able to

1. Solve problems involving personal finances. (DOK 2)
2. Analyze and calculate the amortization of car loans. (DOK 2-4)
3. Determine the amount of the down payment for a mortgage, the monthly mortgage, and the total interest on a mortgage. (DOK 2)
4. Analyze the effects of interest rates, down-payments, and borrowed amount on loan. (DOK 2-3)
5. Analyze data displayed in print and internet media. (DOK 4)
6. Read and interpret tables and bar graphs. (DOK 2)
7. Organize data with frequency tables, dot plots, histograms and stem and leaf plots. (DOK 2)
8. Determine measures of central tendencies (mean, median, mode). (DOK 1)
9. Determine measures of variability including range and standard deviation. (DOK 1)
10. Solve problems in context using geometric models, distinguishing between problems requiring area versus perimeter. (DOK 3)
11. Differentiate between similar and congruent geometric figures. (DOK 3)
12. Solve problems involving similar figures. (DOK 3)
13. Apply the Pythagorean theorem to solve real world problems. (DOK 2)
14. Identify the sides and corresponding angles of a right triangle. (DOK 1)
15. Determine the lengths of the sides of similar right triangles using proportions. (DOK 2)
16. Apply trigonometric Ratios to solve real-world problems. (DOK 3)
17. Write formulas for and calculate surface areas of rectangular prisms, right circular cylinders, and spheres. (DOK 2)
18. Write formulas for and calculate the volume of three-dimensional figures. (DOK 1)
19. Apply geometry formulas to solve problems and include the use of scale drawings in the problem-solving process. (DOK 2)

**Core Activities and Corresponding Instructional Methods:**

1. Develop students' ability to find and make predictions based on measures of central tendency and variation. Analyze visual displays of data sets.
    a. Instructional Methods:
        i. Diagnostic questioning
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
    b. Core Activities:
        i. Delaware River Flow Rate Project- Each student will be given a month's data on the flow rate of the Delaware River. Students will use the functions on excel to find and interpret the measures of central tendency, display their data, and use conditional formatting while organizing and analyzing the data.
2. Expose and develop students to possible personal finance decisions that will impact their future.

a. Instructional Methods:
    i. Diagnostic questioning
    ii. Cooperative learning groups
b. Core Activities:
    i. Budget Project - Students will plan for their future education, career, savings, living situation, and transportation. Students will create a budget sheet using google sheets to track their expenses and income during this simulation.
    ii. Vacation Plans- Given a budget, students will have to plan a future vacation including flights, hotel, transportation, food, and activities. Students will complete research to find the average temperature and rainfall amount of their chosen destination during the month they are planning to visit.

3. Develop students' ability to analyze financial options by calculating and comparing loan options.
    a. Instructional Methods:
        i. Diagnostic questioning of loans
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
    b. Core Activities:
        i. Which car loan would you rather? - Given different loan amounts, down payments, rates, and credit scores, students will use an amortization calculator to determine which car loan makes the most sense for their financial status.
        ii. Amortization Schedule with Extra Payment- Google Sheets Template
4. Develop students' ability to solve problems using similarity, the Pythagorean Theorem or trigonometry.
    a. Instructional Methods:
        i. Diagnostic questioning and exploration of right triangles
        ii. Direct instruction using Smart Technology
        iii. Guided practice
        iv. Cooperative learning groups
    b. Core Activities:
        i. Indirect Measures Project- Students will find a tall object in or around the school. Students will use what they know about similarity and trigonometry to estimate the height of the object. Students will use similar figures, their height and shadow, to estimate the height of the object or students will use trigonometry to find the height of the object by creating an inclinometer using a protractor.
5. Expose students' prior knowledge of the perimeter and area of polygons, including the area and circumference of circles.
    a. Instructional Methods:
        i. Diagnostic questioning and exploration of area and perimeter
        ii. Direct instruction using Smart Technology
        iii. Guided practice

iv. Cooperative learning groups

6. Develop students' ability to apply their knowledge of surface area and volume in real-world application.
    a. Instructional Methods:
        i. Direct instruction using Smart Technology
        ii. Guided practice
        iii. Cooperative learning groups
    b. Core Activities:
        i. Flow Rate- Students will be given a 3-Dimensional object to fill with water (fish tank, swimming pool, ext.). Students will find the volume of their object and use flow rates to estimate the length in time it will take for their 3-Dimensional figure to be filled.
7. Weekly Cumulative Assessments

**Assessments:**

- **Diagnostic:**
    - Vertical whiteboards and in small groups
    - Teacher questioning and observation
- **Formative:**
    - Teacher observations, questions, discussions
    - Individual and group practice
    - Teacher prepared projects
    - Teacher Prepared Weekly Cumulative Assessments
- **Summative:**
    - Teacher Prepared Weekly Cumulative Assessments

**Extensions:**

- Scale Drawings Project- Given a budget, students will remodel a master bedroom. Students will use the internet to find furniture for their bedroom. Students will find and price new floors and paint for the walls. Students will create a blue-print of their remodeled bedroom using what they know about similar figures and scale factors.
- Flow Rate Project: Students can compare and contrast how long it will take to fill up the prisms given different flow rates. (garden hose, sink spout, fire truck hose)

**Correctives:**

- Remediation: IXL and/or Delta math
- Desmos Activities
- Individualized Kuta Practice